

# Probing the twist-3 multi-gluon correlation functions by $p^\uparrow p \rightarrow DX$

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**Abstract.** We study the single spin asymmetry (SSA) for the  $D$ -meson production  $A_N^D$  in the  $pp$  collision,  $p^\uparrow p \rightarrow DX$ , in the framework of the collinear factorization. Since the charm quark is mainly produced through the  $c\bar{c}$ -pair creation from the gluon-fusion process, this is an ideal process to probe the twist-3 triple-gluon correlation functions in the polarized nucleon. We derive the corresponding cross section formula for the contribution of the triple-gluon correlation function to  $A_N^D$  in  $p^\uparrow p \rightarrow DX$ , applying the method developed for  $ep^\uparrow \rightarrow eDX$  in our previous study. As in the case of  $ep^\uparrow \rightarrow eDX$ , our result differs from a previous study in the literature. We will also present a simple estimate of the triple-gluon correlation functions based on the preliminary data on  $A_N^D$  by RHIC.

## 1. Introduction

The single spin asymmetry (SSA) in inclusive hard processes appears as a twist-3 observable in the collinear factorization which is valid in describing the large- $P_T$  hadron productions. In this twist-3 mechanism, SSA is represented in terms of multi-parton correlations in the hadrons. Such correlations in the transversely polarized nucleon can be represented by the quark-gluon correlation functions and the triple-gluon correlation functions. So far the former effect has been investigated for various processes. To probe the latter effect, open-charm production in  $ep$  and  $pp$  collisions is an ideal tool, since the  $c\bar{c}$  pair is created mainly by the gluon-photon or gluon-gluon fusion processes.

In our recent paper [1], we formulated a method of calculating the contribution of the triple-gluon correlation functions to the single-spin-dependent cross section for the  $D$ -meson production in semi-inclusive deep inelastic scattering (SIDIS),  $ep^\uparrow \rightarrow eDX$ . There we identified the complete set of the relevant triple-gluon correlation functions and derived the corresponding cross section in the leading order with respect to the QCD coupling constant. Our result differed from the previous study in the literature [2], and we have clarified the origin of the discrepancy.

In this work, we will apply the formalism of [1] to the  $pp$ -collision,  $p^\uparrow p \rightarrow DX$ , and derive the corresponding single-spin-dependent cross section. This study is relevant to the ongoing RHIC experiment. We also provide a simple estimate for the constraint on the triple-gluon correlation functions, using a preliminary data on  $p^\uparrow p \rightarrow DX$  reported at RHIC [3].

## 2. Triple-gluon correlation functions for the transversely polarized nucleon

Triple-gluon correlation functions in the transversely polarized nucleon are defined as the correlation functions of the three gluon's field strength tensor  $F^{\alpha\beta}$ . Since there are two ways of constructing color-singlet correlation functions from the contraction with the symmetric and antisymmetric structure constants for the color SU(3) group,  $d_{bca}$  and  $f_{bca}$ , one can define two gauge-invariant correlation functions in the nucleon as [1]

$$\begin{aligned} O^{\alpha\beta\gamma}(x_1, x_2) &= -g(i)^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS | d_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= 2iM_N \left[ O(x_1, x_2) g^{\alpha\beta} \epsilon^{\gamma p n S} + O(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha p n S} + O(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta p n S} \right], \end{aligned} \quad (1)$$

$$\begin{aligned} N^{\alpha\beta\gamma}(x_1, x_2) &= -g(i)^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)} \langle pS | i f_{bca} F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | pS \rangle \\ &= 2iM_N \left[ N(x_1, x_2) g^{\alpha\beta} \epsilon^{\gamma p n S} - N(x_2, x_2 - x_1) g^{\beta\gamma} \epsilon^{\alpha p n S} - N(x_1, x_1 - x_2) g^{\gamma\alpha} \epsilon^{\beta p n S} \right], \end{aligned} \quad (2)$$

where  $M_N$  is the nucleon mass,  $S$  is the transverse-spin vector for the nucleon,  $n$  is the light-like vector satisfying  $p \cdot n = 1$  and we used the shorthand notation as  $F^{\beta n} \equiv F^{\beta\rho} n_\rho$  etc. Hermiticity,  $PT$ -invariance and permutation symmetry lead to the decomposition of (1) and (2) in terms of the two real functions  $O(x_1, x_2)$  and  $N(x_1, x_2)$  which have the following symmetry properties,

$$\begin{aligned} O(x_1, x_2) &= O(x_2, x_1), & O(x_1, x_2) &= O(-x_1, -x_2), \\ N(x_1, x_2) &= N(x_2, x_1), & N(x_1, x_2) &= -N(-x_1, -x_2). \end{aligned} \quad (3)$$

The gauge-link operator which restores gauge invariance of the correlation functions is suppressed in (1) and (2) for simplicity.

## 3. Polarized cross section formula for $p^\uparrow p \rightarrow DX$

The formalism for calculating the contribution of the triple-gluon correlation functions to  $ep^\uparrow \rightarrow eDX$  developed in [1] can be directly applied to  $p^\uparrow p \rightarrow DX$  [4]. In this process, a  $c\bar{c}$ -pair is created by the gluon-fusion process, and the initial-state-interaction (ISI) diagrams as well as the final-state-interaction (FSI) diagrams give rise to the single-spin-dependent cross section as a pole contribution at  $x_1 = x_2$ . The twist-3 cross section for  $p^\uparrow(p, S_\perp) + p(p') \rightarrow D(P_h) + X$  with the center-of-mass energy  $\sqrt{s}$  is given by

$$\begin{aligned} P_h^0 \frac{d\sigma^{3\text{-gluon}}}{d^3P_h} &= \frac{\alpha_s^2 M_N \pi}{s} \sum_{f=c\bar{c}} \int \frac{dx'}{x'} G(x') \int \frac{dz}{z^2} D_f(z) \int \frac{dx}{x} \delta(\tilde{s} + \tilde{t} + \tilde{u}) \epsilon^{p_c p n S_\perp} \frac{1}{\tilde{u}} \\ &\times \left[ \delta_f \left\{ \left( \frac{d}{dx} O(x, x) - \frac{2O(x, x)}{x} \right) \hat{\sigma}^{O1} + \left( \frac{d}{dx} O(x, 0) - \frac{2O(x, 0)}{x} \right) \hat{\sigma}^{O2} + \frac{O(x, x)}{x} \hat{\sigma}^{O3} + \frac{O(x, 0)}{x} \hat{\sigma}^{O4} \right\} \right. \\ &\left. + \left\{ \left( \frac{d}{dx} N(x, x) - \frac{2N(x, x)}{x} \right) \hat{\sigma}^{N1} + \left( \frac{d}{dx} N(x, 0) - \frac{2N(x, 0)}{x} \right) \hat{\sigma}^{N2} + \frac{N(x, x)}{x} \hat{\sigma}^{N3} + \frac{N(x, 0)}{x} \hat{\sigma}^{N4} \right\} \right], \end{aligned} \quad (4)$$

where  $\delta_c = 1$  and  $\delta_{\bar{c}} = -1$ ,  $D_f(z)$  represents the  $c \rightarrow D$  or  $\bar{c} \rightarrow \bar{D}$  fragmentation functions,  $G(x')$  is the unpolarized gluon density, and  $p_c$  is the four-momentum of the  $c$  (or  $\bar{c}$ ) quark (mass  $m_c$ ) fragmenting into the final  $D$  (or  $\bar{D}$ ) meson. The partonic hard cross sections in (4) are given by

$$\begin{cases} \hat{\sigma}^{O1} &= \left( \frac{1}{C_F} \frac{\tilde{u}-\tilde{t}}{\tilde{s}\tilde{t}\tilde{u}} + \frac{1}{C_F} \frac{\tilde{u}}{\tilde{s}\tilde{t}^2} - \frac{1}{N^2 C_F} \frac{\tilde{s}}{\tilde{t}^2 \tilde{u}} \right) (\tilde{t}^2 + \tilde{u}^2 + 4m_c^2 \tilde{s} - \frac{4m_c^4 \tilde{s}^2}{\tilde{t}\tilde{u}}), \\ \hat{\sigma}^{O2} &= \left( \frac{1}{C_F} \frac{\tilde{u}-\tilde{t}}{\tilde{s}\tilde{t}\tilde{u}} + \frac{1}{C_F} \frac{\tilde{u}}{\tilde{s}\tilde{t}^2} - \frac{1}{N^2 C_F} \frac{\tilde{s}}{\tilde{t}^2 \tilde{u}} \right) (\tilde{t}^2 + \tilde{u}^2 + 8m_c^2 \tilde{s} - \frac{8m_c^4 \tilde{s}^2}{\tilde{t}\tilde{u}}), \\ \hat{\sigma}^{O3} &= \left( \frac{1}{C_F} \frac{\tilde{u}-\tilde{t}}{\tilde{t}^2 \tilde{u}^2} + \frac{1}{C_F} \frac{1}{\tilde{t}^3} - \frac{1}{N^2 C_F} \frac{\tilde{s}^2}{\tilde{t}^3 \tilde{u}^2} \right) (8m_c^4 \tilde{s} - 4m_c^2 \tilde{t}\tilde{u}), \\ \hat{\sigma}^{O4} &= \left( \frac{1}{C_F} \frac{\tilde{u}-\tilde{t}}{\tilde{t}^2 \tilde{u}^2} + \frac{1}{C_F} \frac{1}{\tilde{t}^3} - \frac{1}{N^2 C_F} \frac{\tilde{s}^2}{\tilde{t}^3 \tilde{u}^2} \right) (16m_c^4 \tilde{s} - 4m_c^2 \tilde{t}\tilde{u}), \end{cases} \quad (5)$$

$$\begin{cases} \hat{\sigma}^{N1} &= \left( \frac{1}{C_F} \frac{\tilde{t}^2 + \tilde{u}^2}{\tilde{s}^2 \tilde{t} \tilde{u}} + \frac{1}{C_F} \frac{\tilde{u}}{\tilde{s} \tilde{t}^2} - \frac{1}{N^2 C_F} \frac{\tilde{s}}{\tilde{t}^2 \tilde{u}} \right) \left( \tilde{t}^2 + \tilde{u}^2 + 4m_c^2 \tilde{s} - \frac{4m_c^4 \tilde{s}^2}{\tilde{t} \tilde{u}} \right), \\ \hat{\sigma}^{N2} &= - \left( \frac{1}{C_F} \frac{\tilde{t}^2 + \tilde{u}^2}{\tilde{s}^2 \tilde{t} \tilde{u}} + \frac{1}{C_F} \frac{\tilde{u}}{\tilde{s} \tilde{t}^2} - \frac{1}{N^2 C_F} \frac{\tilde{s}}{\tilde{t}^2 \tilde{u}} \right) \left( \tilde{t}^2 + \tilde{u}^2 + 8m_c^2 \tilde{s} - \frac{8m_c^4 \tilde{s}^2}{\tilde{t} \tilde{u}} \right), \\ \hat{\sigma}^{N3} &= \left( \frac{1}{C_F} \frac{\tilde{t}^2 + \tilde{u}^2}{\tilde{s} \tilde{t}^2 \tilde{u}^2} + \frac{1}{C_F} \frac{1}{\tilde{t}^3} - \frac{1}{N^2 C_F} \frac{\tilde{s}^2}{\tilde{t}^3 \tilde{u}^2} \right) (8m_c^4 \tilde{s} - 4m_c^2 \tilde{t} \tilde{u}), \\ \hat{\sigma}^{N4} &= - \left( \frac{1}{C_F} \frac{\tilde{t}^2 + \tilde{u}^2}{\tilde{s} \tilde{t}^2 \tilde{u}^2} + \frac{1}{C_F} \frac{1}{\tilde{t}^3} - \frac{1}{N^2 C_F} \frac{\tilde{s}^2}{\tilde{t}^3 \tilde{u}^2} \right) (16m_c^4 \tilde{s} - 4m_c^2 \tilde{t} \tilde{u}), \end{cases} \quad (6)$$

where  $N = 3$  and  $C_F = (N^2 - 1)/(2N)$ , and  $\tilde{s}, \tilde{t}, \tilde{u}$  are defined as

$$\tilde{s} = (xp + x'p')^2, \quad \tilde{t} = (xp - p_c)^2 - m_c^2, \quad \tilde{u} = (x'p' - p_c)^2 - m_c^2. \quad (7)$$

The hard cross sections associated with the first term in the first parentheses in (5) and (6) come from ISI, and those associated with the second and the third terms in the same parentheses come from FSI. As in the case of  $ep^\uparrow \rightarrow eDX$ , the cross section in (4) receives the contribution from the four functions  $O(x, x)$ ,  $O(x, 0)$ ,  $N(x, x)$ ,  $N(x, 0)$ . Unlike the case of SIDIS, presence of ISI gives rise to the different hard cross sections for  $O$  and  $N$  functions. From (4), it is clear that the process  $p^\uparrow p \rightarrow DX$  itself is not sufficient for the complete separation of the four functions. For the separation, the process  $ep^\uparrow \rightarrow eDX$  serves greatly, since it has five structure functions with different dependences on the azimuthal angles to which the four functions contribute differently [1].

Our result in (4) differs from a previous work [5]: The result in [5] is obtained from (4) by omitting the terms with  $O(x, 0)$  and  $N(x, 0)$  (and their derivatives) and by the replacement  $O(x, x) \rightarrow O(x, x) + O(x, 0)$  and  $N(x, x) \rightarrow N(x, x) - N(x, 0)$ . This difference originates from an ad-hoc assumption in the factorization formula in [5, 2]. We emphasize the appearance of the four different contributions with  $\{O(x, x), O(x, 0), N(x, x), N(x, 0)\}$  is a consequence of the symmetry property implied in the decomposition (1) and (2), in particular, the different coefficient tensors in front of  $O(x, x)$  and  $O(x, 0)$  (likewise for  $N(x, x)$  and  $N(x, 0)$ ) at  $x_1 = x_2 = x$  lead to different hard cross sections for the above four functions. See [1, 4] for more details.

#### 4. Numerical estimate

As is shown in (4), four nonperturbative functions are involved in the twist-3 cross section for  $A_N^D$ . At present there is no information on these functions. Preliminary data on  $A_N^D$  by RHIC-PHENIX [3] suggests  $|A_N^D| \leq 5\%$ . So we will present a model estimate on the upper bound of these nonperturbative functions for several cases by requiring that the calculated  $A_N^D$  be less than 5%. We first found from (5) and (6) the relations  $|\hat{\sigma}^{O1, O2, N1, N2}| \gg |\hat{\sigma}^{O3, O4, N4, N4}|$  and  $\hat{\sigma}^{O1} \simeq \hat{\sigma}^{O2} \simeq \hat{\sigma}^{N1} \simeq -\hat{\sigma}^{N2}$  for the RHIC kinematics (see below). Accordingly, if  $O(x, x)$  and  $O(x, 0)$  have the same (opposite) sign, they contribute to  $A_N^D$  constructively (destructively). Likewise for  $N(x, x)$  and  $N(x, 0)$ . From this fact we present a model calculation of  $A_N^D$  for two extreme cases:

$$\text{Model 1 :} \quad O(x, x) = -O(x, 0) = N(x, x) = N(x, 0), \quad (8)$$

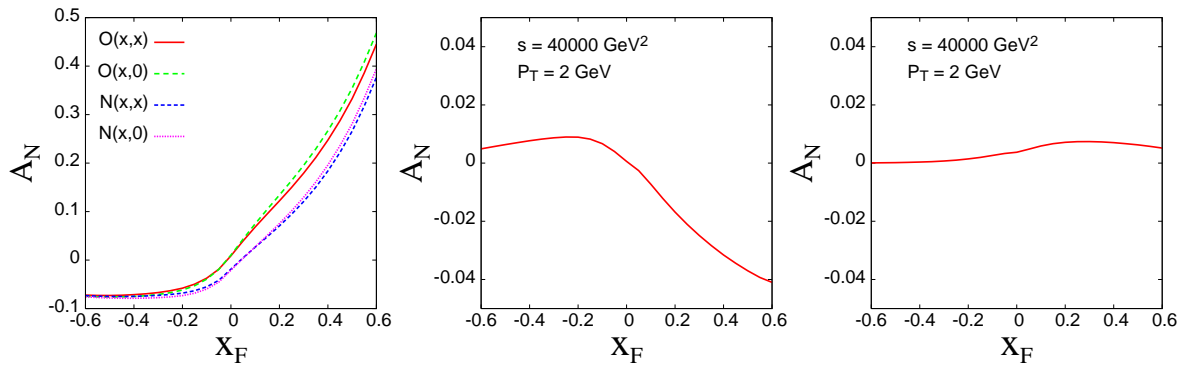
$$\text{Model 2 :} \quad O(x, x) = O(x, 0) = N(x, x) = -N(x, 0). \quad (9)$$

In Model 1 the contributions to  $A_N^D$  from four functions become minimum, while in Model 2 they become maximum. Therefore larger magnitude for the functions is allowed for Model 1 in order to make  $|A_N^D| \leq 5\%$ . For the actual form of the nonperturbative functions, we follow [2] and set

$$O(x, x) = K_G x G(x), \quad (10)$$

where  $G(x)$  is the unpolarized gluon distribution and  $K_G$  is a constant which we determine so as to achieve  $|A_N^D| \leq 5\%$  for the RHIC kinematics. Obviously, the ansatz (10) is a very crude approximation and the result below should be taken only as an estimate of the order-of-magnitude. For the numerical calculation, we use GJR08 distribution [6] for  $G(x)$  and KKKS08 fragmentation function [7] for  $D_f(z)$ . We also assumed the same scale dependence for  $O(x, x)$  *etc* as  $G(x)$  for simplicity. We calculated  $A_N$  for the  $D$  and  $\bar{D}$  mesons at the RHIC energy of  $\sqrt{s} = 200$  GeV and  $P_T = 2$  GeV with the parameter  $m_c = 1.3$  GeV by setting the scale of all the distribution and fragmentation functions at  $\mu = \sqrt{P_T^2 + m_c^2}$ .

Fig. 1 shows the result of  $A_N^D$  for Model 1 with  $K_G = 0.005$ . In order to see the behavior of each term in (4), we showed in the left figure of Fig. 1 the contribution to  $A_N^{D^0}$  from the four functions  $O(x, x)$ ,  $O(x, 0)$ ,  $N(x, x)$  and  $N(x, 0)$ . (For  $N(x, 0)$ ,  $-A_N^D$  is shown.) As is seen from this figure, if one sets all four functions identical, contributions to  $A_N^{D^0}$  from them are very close. In the middle and right figures of Fig. 1,  $A_N$  for the  $D^0$  and  $\bar{D}^0$  mesons are shown, respectively. Even though each of four contribution gives rise to as large as 50 % asymmetry as shown in the left figure, the resulting total  $A_N$  is of  $O(5\%)$  due to the cancellation among them. Therefore if the relation (8) approximately holds,  $K_G = 0.005$  in (10) provides an upper limit for  $O(x, x)$  *etc*. Because of the relation (8), the  $O$ -term and the  $N$ -term contribute to  $A_N$  constructively (destructively) for  $D^0$  ( $\bar{D}^0$ ) meson as shown in Fig. 1. This feature was also observed in [5].



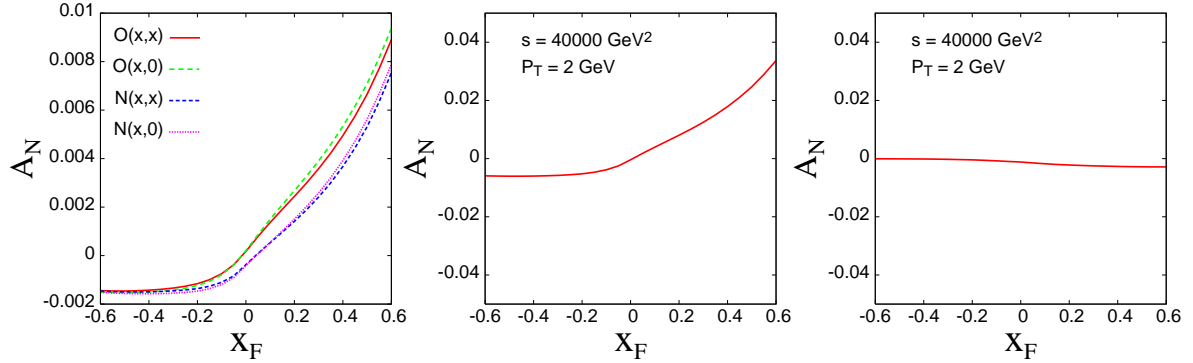
**Figure 1.**  $A_N^D$  for Model 1. Decomposition of  $A_N^D$  into the 4 components (left).  $A_N$  for  $D^0$  meson (middle).  $A_N$  for  $\bar{D}^0$  meson (right).

Fig. 2 shows the result for Model 2 with  $K_G = 0.0001$ . In the left figure of Fig. 2, the behavior of each contribution is shown. In the middle and right figures,  $A_N$  for the  $D^0$  and  $\bar{D}^0$  mesons are shown, respectively. In this Model 2, we had to take  $K_G$  as small as  $K_G \sim 0.0001$ , since  $O(x, x)$  and  $O(x, 0)$  as well as  $N(x, x)$  and  $N(x, 0)$  contribute to  $A_N$  of either  $D^0$  or  $\bar{D}^0$  constructively.

From the left figures of Figs. 1 and 2, one can see that the magnitude of each of the four contributions is very similar if the four functions have the same magnitude. This means that the contributions to  $A_N^D$  from  $O(x, x) + O(x, 0)$  and  $N(x, x) - N(x, 0)$  are, respectively, much larger than those from  $O(x, x) - O(x, 0)$  and  $N(x, x) + N(x, 0)$ , because

$$|\hat{\sigma}^{O1} + \hat{\sigma}^{O2}| \gg |\hat{\sigma}^{O1} - \hat{\sigma}^{O2}|, \quad |\hat{\sigma}^{N1} - \hat{\sigma}^{N2}| \gg |\hat{\sigma}^{N1} + \hat{\sigma}^{N2}|. \quad (11)$$

Since there is no reliable nonperturbative information on these functions, it is natural to expect that all four functions  $O(x, x) \pm O(x, 0)$  and  $N(x, x) \mp N(x, 0)$  have a similar magnitude. In this case,  $A_N^D$  is mostly determined by  $O(x, x) + O(x, 0)$  and  $N(x, x) - N(x, 0)$ , while  $O(x, x) - O(x, 0)$



**Figure 2.**  $A_N^D$  for Model 2. Decomposition of  $A_N^D$  into the 4 components (left).  $A_N$  for  $D^0$  meson (middle).  $A_N$  for  $\bar{D}^0$  meson (right).

and  $N(x, x) + N(x, 0)$  can be neglected by the relation (11). From this observation, one obtains a modest estimate on the upper bound for the combinations as

$$\begin{aligned} |O(x, x) + O(x, 0)| &\leq (0.0003 \sim 0.0004)xG(x), \\ |N(x, x) - N(x, 0)| &\leq (0.0003 \sim 0.0004)xG(x), \end{aligned} \quad (12)$$

if  $|A_N^D| \leq O(5\%)$ . We remind that the relation (11) is a peculiar feature for the process  $p^\uparrow p \rightarrow DX$  and does not generally hold in  $ep^\uparrow \rightarrow eDX$  [1].

## 5. Summary

In this work, we have studied the contribution of the triple-gluon correlation functions to  $A_N^D$  in  $p^\uparrow p \rightarrow DX$ . We derived the corresponding twist-3 single-spin-dependent cross section in the leading order with respect to the QCD coupling constant. The complete cross section receives contribution from the four functions  $O(x, x)$ ,  $O(x, 0)$ ,  $N(x, x)$  and  $N(x, 0)$  as in the case of  $ep^\uparrow \rightarrow eDX$  [1], and differs from the result in a previous work [5]. We have also presented a model calculation for  $A_N^D$  at the RHIC energy. Assuming  $|A_N^D| \leq O(5\%)$  as suggested by the RHIC preliminary data, we obtained a modest estimate (12) for a particular combination of the triple-gluon correlation functions. The detail of our analysis will be reported elsewhere [4].

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